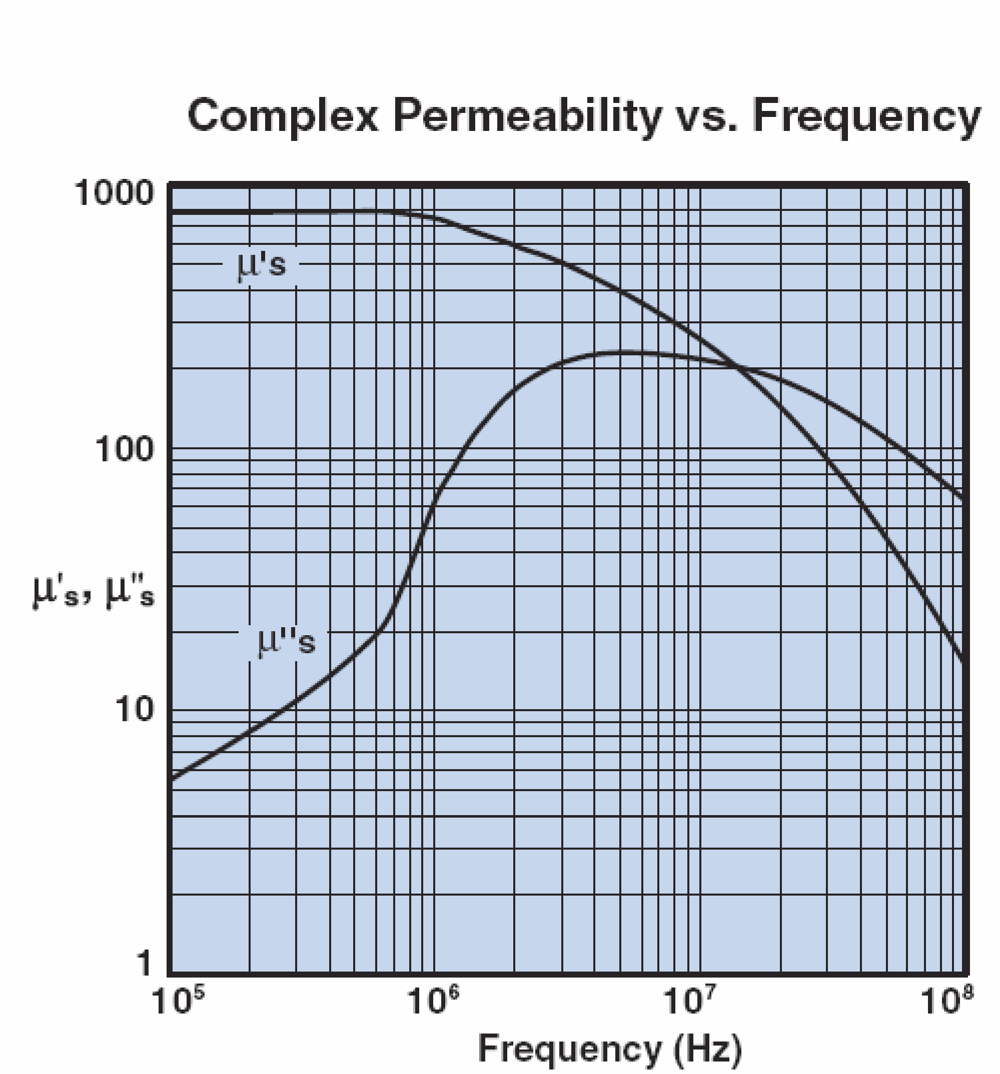
Estimating Q of Inductors with Ferrite Cores from u' and u" Specifications

Written	22 March 2008
Last Revised	25 March 2008

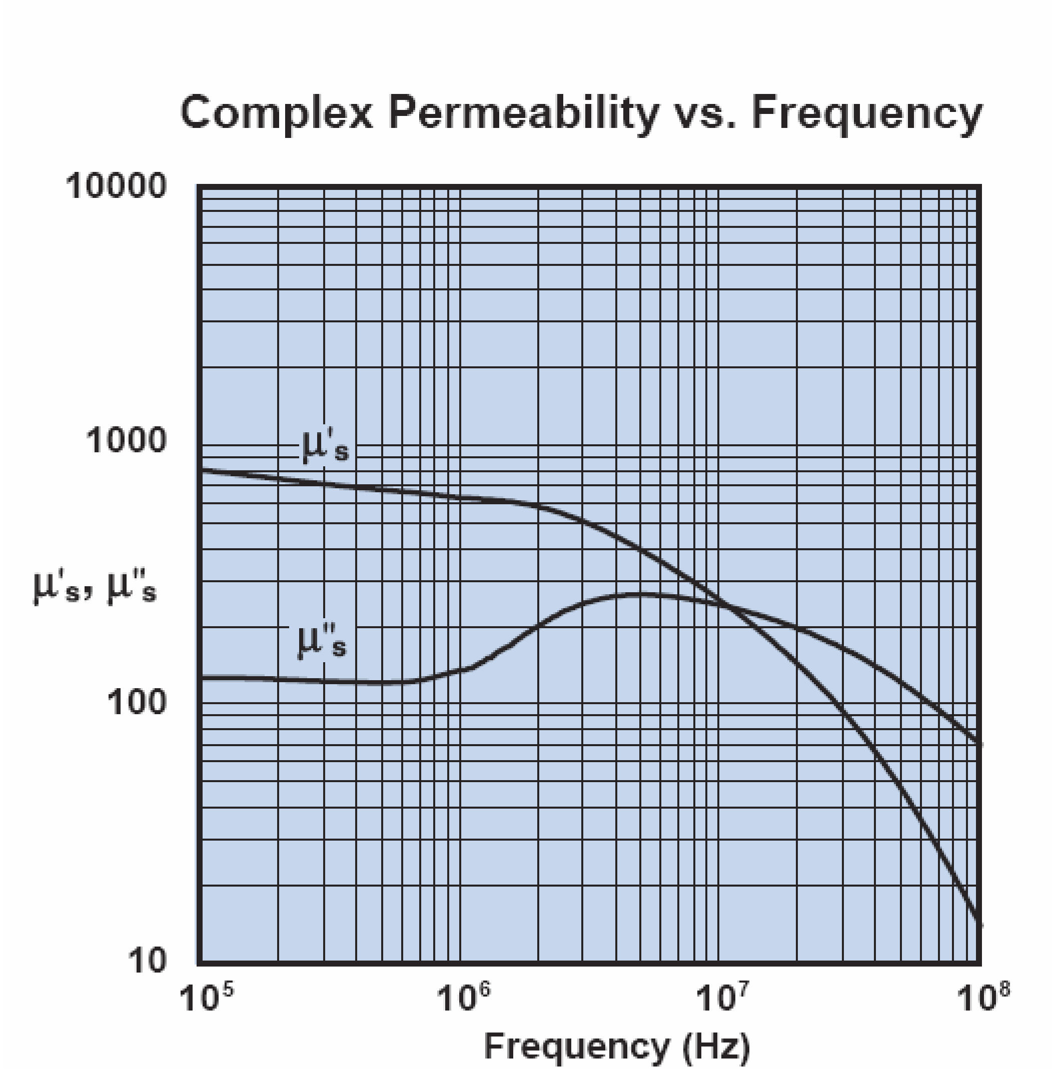
I was asked why an inductor wound on a ferrite core of Fair-Rite 43 material resulted in high loss when used in an 8 MHz filter. The short answer is that type 43 material has very low Q at that frequency, and that this is apparent from the material's characteristic curves. I realized, however, that this might not be obvious unless one has worked through the equations and data.

Let's start with type 43 material's complex permeability characteristics, as illustrated below. The complete datasheet for 43 material can be found at http://www.fair-rite.com/newfair/materials43.htm.

The first curve shows 43 Material data from Fair-Rite's more recent catalogs (starting with Catalog 14, release 3?)



The curve below is from an earlier release of Catalog 14 (undated but appears to be late 1999 or early 2000.) There is a considerable difference in μ" at lower frequencies; decreasing from around 120 to 5.8 at 100 KHz in the newer literature. There's also a slight change in low frequency permeability, 850 in the older catalog and 800 in the newer catalog. In addition to the changes in u'', note that u' remains much flatter up to 1 MHz in the newer data.



I didn't know if this major revision to μ'' represents a reformulation of the 43 material composition, or better test data or is an error, so I asked Fair-Rite whether the newer data reflects changed performance for recently manufactured materials.

I received the following answer from Fair-Rite on 24 March 2008:

> Dear Jack
>
> It is highly likely that the cores you have are "out of date" in terms of the present specification. The online catalog is up to date and shows the present specification for 43 material.
>
> Rgds Alan Keenan

Fair-Rite has made a significant improvement to 43 Material, therefore, by markedly reducing the loss and stabilizing the permeability over a wider frequency range. At the moment, all my 43 material cores seem to be older product, so I can't provide direct comparison measurements until I get my hands on known new products.

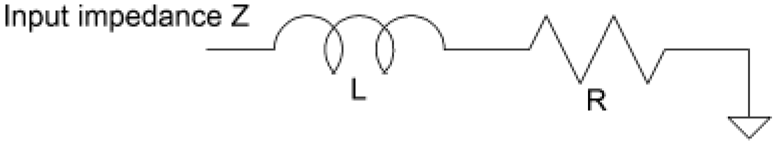
I've covered permeability to some degree at my Type 43 Ferrite B-H Curve page, and you may wish to review that before reading further on this page.

We know that real inductors can be modeled as a perfect inductor with a resistive loss element and a shunting capacitance (turn-to-turn capacitance). The resistive loss can be considered to be in series with the perfect inductor or in parallel with the perfect inductor, with the choice of the model being based upon the circuit being analyzed.

The inductor's loss is from several sources, including:

- Core losses due to hysteresis (see the B-H curve page earlier mentioned)
- Core losses due to eddy currents (generally not a major factor in ferrite cores)
- Copper losses, due to the resistance of the wire. Remember that at radio frequencies, skin effect may greatly increase the resistance over measured DC values
- Dielectric losses in the turn-to-turn capacitance and in the turn-to-core capacitance

The μ factor discussed at my B-H curve page is u'_s in Fair-Rite's complex permeability versus frequency plot. It governs the inductive relationship. μ''_s, on the other hand, governs loss or Q. Let's consider a model that ignores the turn-to-turn capacitance of the inductor (which is generally not a major factor in resistive loss), as shown below. This is the series configuration, where the loss element is modeled as a series resistor.



Considering core losses only, the input impedance Z of the inductor and loss element is

$Z=j\omega L_0 + R = j\omega L_0(\mu_s'-j\mu_s'')$ ohms [1]

L_0 is the inductance of the coil, if wound in air.
ω is the frequency in radians/sec, or 2π times the frequency f in Hz.

If we work through equation [1], remembering that j*j = -1, we find:

$R = \omega L_0 \mu_s''$ [2]
$L = L_0 \mu_s'$ [3]

We also know that the classic definition of an inductor's Q in series form is:

$Q = X_s/R_s$ [4]

Where $X_s = \omega L_s$

With some manipulation, we find $Q = \mu_s' / \mu_s''$ [5]

At 100 KHz, for example, us' is 800 and us'' is 120 using the older catalog data, so we expect to measure an inductor wound on 43 Material as presenting a Q of 800/120 = 6.7, best case, as it neglects other loss sources.

The new performance data places us'' at about 5.5 at 100 KHz, which yields a best case Q of 800/5.5 = 145, a huge improvement over the old catalog data.

One additional factor can be determined from the Complex Permeability plot. If we measure the inductance at frequency f1, we may estimate the inductance at frequency f2 by taking the ratio of the two μ_s' values.

For example, at 100 KHz, the measured inductance is 10 µH. What is the approximate inductance at 1 MHz?

Using the older data, at 100 KHz, u' is approximately 800, whilst at 1 MHz it's 625. Hence, the true inductance at 1 MHz is 10 µH * 625 / 800 = 7.8 µH.

Measured Data

To determine how well the curves match test samples, I wound three inductors on the following cores:

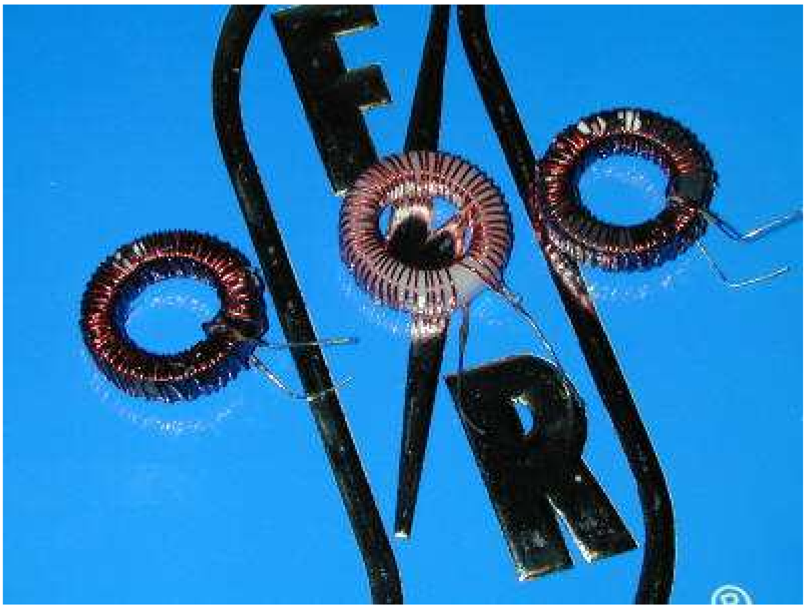
- Fair-Rite 43 material, PN 5943001001
- Fair-Rite 61 material, PN 5961001001
- Delrin core, same dimensions as Fair-Rite 59xx001001

All three inductors have 50 turns of no. 24 AWG enamel magnet wire.

Three test cores on top of a Fair-

Rite catalog

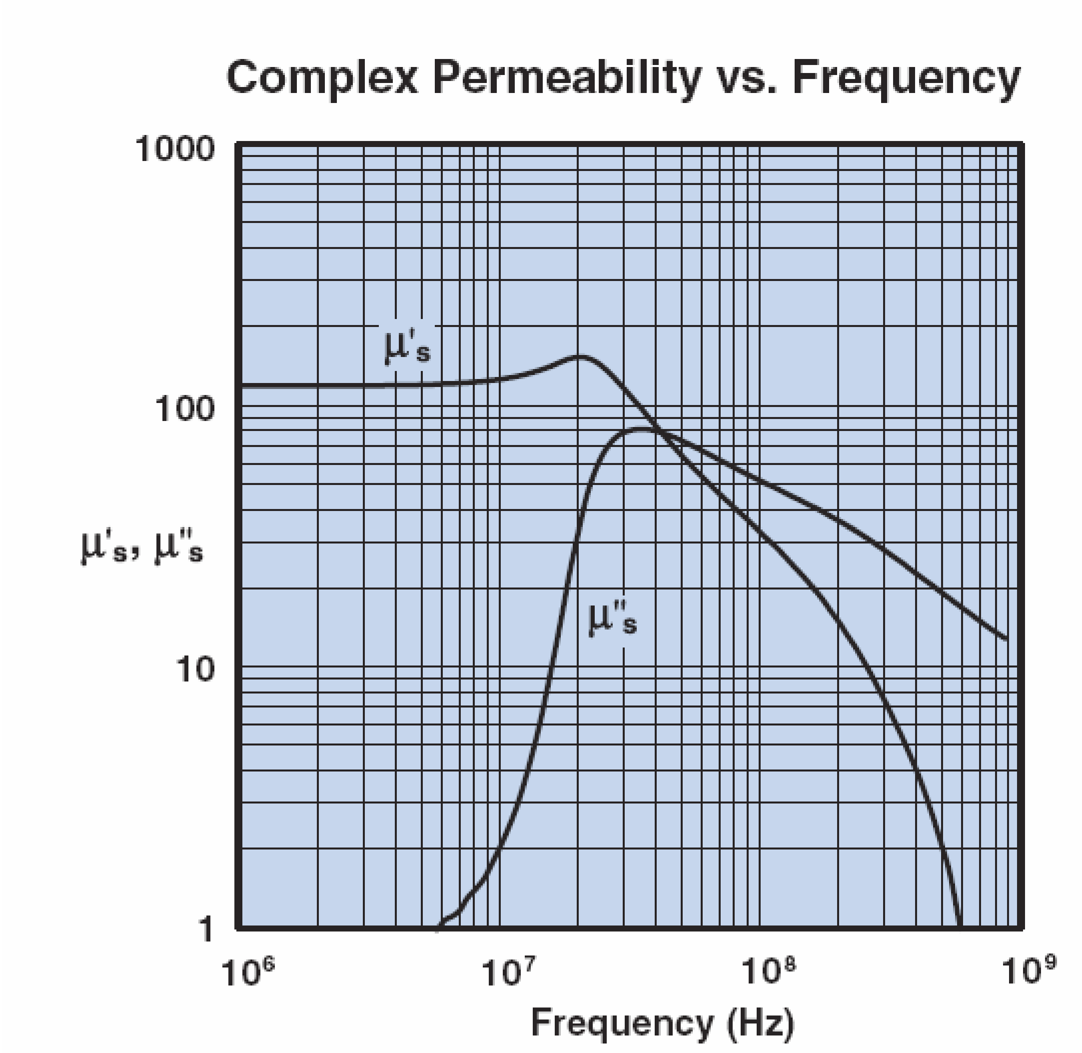
Wound coils



The core parameters are:

Mat'l	OD inch	ID inch	Thick inch	C1 cm^{-1}	le cm	Ae cm^2	Ve cm^3	AL nH
Delrin	1.142	0.748	0.295	19.8	7.3	0.37	2.70	--
43	same	same	same	same	same	same	same	540
61	same	same	same	same	same	same	same	80

61 Material has the following Complex Permeability curve. Curves for this material seem unchanged from 2000 to the present catalog.



First, we'll compute the theoretical inductance. For a toroid, it can be shown that the inductance if wound on a material with relative permeability 1, such as air or Delrin, L_0, is:

$L_0 = 4\pi N^2 x10^{-9}/C1$ Henries [6]

Substituting the values for N and C1, the inductor wound on a Delrin core has a theoretical inductance of 1.59 μH.

To compute inductance from the AL value, simply multiply it by the square of the number of turns:

$L = N^2 A_L$ [7]

Applying equation [7], we determine that the inductor wound on 43 Material core has a theoretical inductance of 1.35 mH and the 61 Material inductor has a theoretical inductance of 200 μH.

We can also compute the inductance and estimate the maximum inductor Q of the two ferrite cores using the approach in Equations [1] through [5], based on L_0's calculated 1.59 μH.

Taking the data from Fair-Rite's graphs (older graph for 43 Material) at the test frequencies, we can calculate the inductor parameters as:

Material	Frequency KHz	L0 uH	u'	L uH	u"	R ohms	Q
43	1	1.59	800	1272	120	0.1908	6.7
43	250	1.59	710	1129	120	47.7	5.9
61	1	1.59	125	199	<1	0.00159	>125.0
61	790	1.59	125	199	<1	1.2561	>125.0

The Q values for Type 61 material are in fact greater than the computed numbers because the data sheet does not show u" values below 1.0. The data suggests that the true u" values are well below 1.

Let's see how the measured values stack up. Data at 1 KHz is taken with a General Radio 1658 Digibridge, with the remaining data taken with an HP 4342A Q-meter.

Material	Frequency KHz	Measured L uH	Measured Q
43	1	2241	7.31
43	250	1460	6.1
61	1	188.14	0.93
61	790	188.5	294
Delrin	1	2.28	0.12
Delrin	7900	2.12	108

Looking at the 43 Material data first, we see the measured inductance at 1 KHz is considerablyy greater than predicted using L_0 and the permeability curves. There is an assumption here, however, that the permeability factor of 800 remains constant below 100 kHz, the lowest frequency point in the data curve. Fair-Rite's newer data curve suggests this is the case, but the older data curve strongly suggests that µ' and hence L, continues to increase as the frequency drops below 100 KHz. If this is the case, then the measured 2241 µH makes much more sense.

At 250 KHz, the measured inductance is considerably closer to the measured value. The estimated Q matches the measured data rather well, both at 1 KHz and 250 KHz. Computing L using the published A_L value (which, of course, is only valid so long as the frequency is below 100 KHz where presumably u' is 800) produces reasonable match to the measured data at 250 KHz.

The 61 Material also has two missing data elements. First, like 43 Material, the lowest frequency information is for 100 KHz. And, µ" is not shown below 1, which excludes both the 790 KHz test frequency and 1 KHz. With respect to inductance, the measured 188 µH is reasonably close to both the 199 µH predicted using L_0 and the 200 µH based upon AL values.

At 790 KHz, the measured Q certainly is > 125, measuring nearly 300. This suggests that at this frequency µ" is in the range of 0.25 or less, considering that the measured Q includes all loss factors, not just the core loss.

At 1 KHz, however, we find the measured Q is wildly different than the predicted Q. The reason for this is not hard to find. At 1 KHz, 188 µH inductance has a reactance of approximately 1.2 ohms. The inductor is wound with 48 inches of No. 24 AWG magnet wire, with a resistance of 25.7 ohms per 1000 ft. resulting in 0.103 ohms series resistance from copper loss alone. This places a ceiling on Q of around 10, without regard for other losses.

The Delrin core also measures quite a bit higher than the calculated value. At 1 KHz, a 2.28 µH inductor has a reactance of 0.014 ohms. With 0.103 ohms of resistance from the copper wire, the expected Q will not exceed 0.14, which is, in fact, quite close to the measured 0.12. At 7.9 MHz, the reactance is 105 ohms, which yields a maximum Q considering copper loss only of around 1000. However, increased resistance due to skin effect and other practical considerations must be considered when Q is estimated at radio frequencies. In fact, skin effect at 7.9 MHz increases the effective winding resistance to around 4 ohms, which reduces the maximum expected Q to 250 or less. Other losses, of course, will push the expected Q even lower.